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Optimal placement of actuators for active vibration control of seismic excited tall buildings using a multiple start guided neighbourhood search (MSGNS) algorithm

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Abstract

Active control devices can be implemented on seismically excited high rise buildings using appropriate active control theory, to reduce structural responses to a desired level. Certain locations of the structure are advantageous for placement of actuators in the sense that these locations effectively reduce the structural responses. Hence, optimal placement of actuators at discrete locations is an important problem that will have significant impact on control of civil structures like high rise buildings, bridges, etc. This optimal placement problem leads to a combinatorial optimisation and is difficult to solve.

This paper presents a multi-start meta-heuristic algorithm called multiple start guided neighbourhood search (MSGNS) algorithm, which makes use of the good features of guided local searches like simulated annealing (SA) and tabu search (TS). Four distinct design criteria which influence the active control design are considered in this paper to study the optimal actuator placement problem. The sensitivities of the four optimisation criteria with respect to different earthquake records are explored. Further, in this paper, we deviate from the usual practice of using shear building models (or simple lumped mass model) in active control research for finding optimal actuator locations. Instead, we use detailed finite element models and demonstrate through numerical examples their effectiveness in arriving at the optimal actuator locations. Finally, the superior performance of the proposed MSGNS algorithm over popular meta-heuristic algorithms like GA, SA and TS is demonstrated through numerical experiments.

1. Introduction

Civil engineering structures, particularly high-rise buildings are especially susceptible to random vibrations due to ground acceleration or due to high wind forces. Recent advancements in the field of structural control have significantly reduced the probability of catastrophic failure of high rise buildings and other civil engineering structures during earthquakes. Several active control techniques have recently been developed as a possible way of reducing the vibrations of civil engineering structures during seismic excitations or strong wind gusts. These active control systems have been devised using system control theory concepts, which make use of

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structural response information in manipulating the values of a set of control forces acting on the building structures. Different approaches have been used in designing the foregoing control systems. These include, optimal control theory [1,2], sliding mode control [3] and robust control techniques [4], etc.

Active control devices can therefore help in considerably improving the safety and serviceability of the seismically excited tall buildings. Many devices and mechanisms have been introduced and some of the most commonly used devices include: active tuned mass dampers and active tendon mechanisms (ATM) [5,6]. Active tuned mass damper is one of the most popular active control device presently being used. They have been extensively used and tested in many tall structures in Japan, New York and Boston and found to be effective in reducing the response of high-rise buildings excited by high winds and earthquakes [7]. However, the sheer size of these devices makes them extremely difficult to accommodate on any other floor except on roof top. On the other hand, ATM [8,9] gives much more flexibility in control and can easily be installed at discrete locations, i.e., on any floor or bay of the high rise building structures. Moreover, unlike active tuned mass dampers, it is possible to control other modes rather than the fundamental mode using ATM. This can cause a greater reduction in the velocity and acceleration. Therefore, these ATMs are being extensively used for retrofitting of the existing structures. In view of this, the present work uses the ATMs.

It is well known that certain locations of the structure are advantageous for placement of the controller in the sense that these locations effectively reduce structural response while using the minimum control effort. Hence, there is a greater significance in optimising the placement of actuators on actively controlled structures, like tall buildings subjected to seismic excitation. Most of the earlier works attempt to use either genetic algorithms (GA) or simulated annealing (SA) to solve the nonlinear discrete optimisation problem. However, motivated by the fact that there is clear edge in obtaining quality optimal solutions by using customised guided local search algorithms rather than using generic optimisation algorithms like GA, we propose a new multiple start algorithm called multiple start guided neighbourhood search (MSGNS) algorithm for investigating the combinatorial optimisation problem of optimal placement of limited number of actuators on tall buildings.

1.1. Previous relevant work

Several researchers have earlier attempted the problem of optimal actuator positioning. Chen et al. [10] and Onoda and Hanawa [11] employed SA to solve the configuration problem of actuators. Rao and Pan [12] have used GA to solve discrete optimal actuator location selection problem and claimed that their approach produce global or near global optimal solution. Furuya and Haftka [13] used GA to study the problem of placing actuators on space structures. Liu et al. [14] investigated the integrated optimisation of adaptive structural topology and actuator locations using SA. Mohamed and Roorda [15] investigated the problem of optimum control configuration that maximises the control effectiveness and minimises the control cost for a bridge-like structure with active tendons. Fang et al. [16] and Li et al. [17] employed GA in the design of structural control system. Sadri et al. [18] adopted GA to study the optimal locations of actuators in predicting the closed-loop frequency response of a plate for active vibration control. Hiramoto et al. [19] considered the optimal placement of sensors and actuators for an undamped simply supported beam with the controller formulated as an H_∞ controller based on normalised coprime factorisation. The optimal placement problem is solved using a quasi-Newton optimisation method. Li et al. [20] considered the optimisation of the number and location of control actuators using a 3 level algorithm: one level considers the optimal control law, the second level considers the configuration of the actuators, and the final level considers the number of actuators. The optimal number and location of actuators for a 16-storey building is considered using Linear Quadratic Regulator (LQR) theory as the control strategy. A two level GA is used to perform the optimisation. Liu et al. [21] studied the effect of 18 different earthquake excitations on the optimal placement of actuators to minimise the maximum displacement of the top floor and found that the placement is not influenced by the earthquake level. Amini and Tavassoli [22] have used a trained neural network to arrive at optimal number and placement of controllers on a 12-storey shear building model. However, little effort has been made so far on optimal placement of restricted number of actuators and with multiple optimisation criteria.

1.2. Present work

The proposed work differs from the earlier in the following issues:

It can be observed from the literature that in most of the control applications developed so far, the structures have been modelled using a simple lumped mass shear building model. The main advantage of this approach is its simplicity and numerical efficiency because only one degree of freedom is assigned to each floor. However, the shear building model is based on the assumption that the floor beams are perfectly rigid. It can be shown that significant error in natural frequencies of the frame occurs when the floor beams are not much stiffer than the columns [23]. Moreover, the shear building model does not provide detailed information such as the distribution of displacement and stresses in the building frame which are of interest to the structural designers. To complement the simple shear building model, finite element modelling provides additional flexibility and accuracy as well as detailed knowledge of displacements and stresses. In the context of optimal placement of actuators, the additional disadvantage with shear building models is that it cannot precisely indicate the positioning of actuators with respect to the specific bay of the multiple bay framed structures. In the present work, we use detailed finite element models to compute peak controlled parameters (fitness function evaluation) and has been combined with the proposed MSGNS algorithm to optimise the placement of actuators.

Most of the earlier research concentrates on a specific objective to devise the optimal placement algorithms. However, the proposed work examines four different essential requirements in active vibration control, and their influence in arriving at the optimal positioning of actuators.

Most of the earlier works uses either GA or SA to devise the optimal placement algorithms. However, the GA is based on population-based concept and the number of function evaluations required for convergence will be larger. Even though, SA deals with single solution, the cooling schedule is difficult to set in order to obtain optimal solutions and also simultaneously maintain good computational efficiency. A stringent cooling schedule results in very large number of function evaluations and a rather relaxed convergence criteria often results in sub-optimal solutions. Since each function evaluation in the present work involves detailed dynamic finite element analysis of large framed structures using time integration, the computational cost is very high. Hence it is desirable to formulate a meta-heuristic algorithm which can provide optimal solutions to the present combinatorial problem with least number of function evaluations. Keeping these things in view, we propose a meta-heuristic algorithm called MSGNS algorithm which makes use of the best features of the guided search algorithms like SA and tabu search (TS) and at the same time overcomes most of the traditional short comings of SA. Apart from this, the proposed algorithm is built with customised neighbourhood search algorithms. In order to prevent the recycling of already visited solutions, TS is embedded in the proposed multiple start algorithm. We also present results obtained by other popular meta-heuristic algorithms, i.e., GA, SA and also TS to demonstrate the superiority of the proposed MSGNS algorithm both in terms of quality of solution obtained and also the computational performance.

We also demonstrate in the paper, the clear differences in optimal actuator positioning using simplified shear building model and detailed finite element model through numerical simulation studies to further emphasise our stand on using more sophisticated structural modelling for optimal placement of actuators.

Finally, we also examine the sensitivity of the earthquake characteristics on the optimal positioning of actuators of all the four distinct objectives considered in this paper.

In the present work, the LQR algorithm is employed for active control of structures. The optimisation problem of locating optimal positions of given number of actuators has been formulated as a combinatorial problem.

2. Active control algorithm

An active controlled structure with n degrees of freedom is subjected to earthquake excitation $w(t)$ and encountered by the control forces $u(t)$. The governing equation of motion for the structure can be expressed as

$$M\ddot{x}(t) + C\dot{x}(t) + Kx(t) = B_1 u(t) + E_1 w(t), \tag{1}$$

where M, C and K are $n \times n$ mass, damping and stiffness matrices, respectively, $x(t)$ is a $n \times 1$ displacement vector. B_1 is a $n \times m$ matrix defining the location of m actuators.

Eq. (1) can be represented in state space form as

$$\dot{z}(t) = A_c z(t) + B_c u(t) + E_c w(t), \tag{2}$$

where

$$z(t) = \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix}, \quad A_c = \begin{bmatrix} 0 & 1 \\ -M^{-1}K & -M^{-1}C \end{bmatrix}, \tag{3}$$

$$B_c = \begin{bmatrix} 0 \\ M^{-1}B_1 \end{bmatrix}, \quad E_c = \begin{bmatrix} 0 \\ M^{-1}E_1 \end{bmatrix}. \tag{4}$$

Eq. (2) represents a linear and time-invariant control system. The active control force can be found subjected to the condition that the quadratic objective function is minimised:

$$J(t) = \tfrac{1}{2} \int_0^{t_f} [z(t)^{\mathrm{T}} Q z(t) + u(t)^{\mathrm{T}} R u(t)]\, \mathrm{d}t, \tag{5}$$

where, Q is a $2n \times 2n$ positive semi-definite weighting matrix. R is a $m \times m$ symmetric positive definite weighting matrix for the control force.

Applying the optimal control theory and assuming that the control force vector $u(t)$ is generated by feedback of the state vector $x(t)$ and $\dot{x}(t)$ alone, that is

$$u(t) = -R^{-1} B_c^{\mathrm{T}} P z(t), \tag{6}$$

where P is Riccati matrix that can be obtained by solving the Riccati matrix equation:

$$PA_c + A_c^{\mathrm{T}} P - PB_c R^{-1} B_c^{\mathrm{T}} P + Q = 0. \tag{7}$$

Substituting Eq. (7) in to Eq. (1), we get

$$M\ddot{x}(t) + (C + G_C)\dot{x}(t) + (K + G_K)x(t) = F(t), \tag{8}$$

where $G_C = B_1 R^{-1} B_1^{\mathrm{T}} M^{-1} P_4$ is the damping gain and $G_K = B_1 R^{-1} B_1^{\mathrm{T}} M^{-1} P_3$ is stiffness gain. Eq. (8) can be solved using Newmark method [24].

3. Formulation of objective function

The prime objective of active vibration control of tall buildings is to

(i) minimise the peak lateral displacement, $X_{\max}$,
(ii) minimise the peak inter-storey drift ratio, defined as $(X_i - X_{i-1})_{\max}/h_i$, where h_i represents the height between the ith and $(i-1)$th stories,
(iii) minimise the peak absolute lateral acceleration, $(a + a_g)_{\max}$,
(iv) optimise the average control force requirement defined as

$$\frac{1}{t} \int_{t_i}^{t_f} (u^{\mathrm{T}} u)^{1/2}\, \mathrm{d}t.$$

Among these parameters considered, (i) and (ii) are directly related to structural safety, and (iii) serves as an indicator of human reactive feeling. The fourth objective related to control force, reflects the efficiency of the installed active control system. The influence of these four parameters is considered in the present work to arrive at optimal placement of actuators.

As mentioned earlier, the problem of optimal placement of the actuators has been formulated as a combinatorial optimisation problem. The design variables are the positions of the actuators on the structure.

Each potential actuator location on the structure is represented by a design variable P_i, which can either be zero or one. For a typical framed structure, the potential actuator locations are all the bays in the frame, equal to number of floors (NF) × number of bays in each floor (NB). If the actuator is present at any typical location, i, then P_i will be assigned a value equal to 1 otherwise $P_i = 0$ and $N = \sum_{i=1}^{\mathrm{NBXNF}} P_i$, where N is the number of actuators chosen by the user.

The objective function can be any one of the following depending upon the optimisation criterion chosen. The mathematical formulations of the objective functions can be described as follows.

(a) Peak controlled lateral displacement

$$\min_p O(N,P) = \min_p \max|x_j(t_i)| \quad \text{where } 1 \leqslant j \leqslant q, \quad 1 \leqslant i \leqslant h, \tag{9}$$

and $P = \{P_1, P_2, P_3, \ldots, P_{NT}\}$.

$$\mathrm{NT} = \text{number of floors (NF)} \times \text{number of bays (NB) and } \sum_{i=1}^{NT} P_i = N.$$

Subjected to the design constraints

$$\text{(i)} \quad O(N,P) \leqslant x_P, \tag{10a}$$

$$\text{(ii)} \quad \max|D_j(t_i)| \leqslant D_P, \tag{10b}$$

$$\text{(iii)} \quad \max|U_j^l(t_i)| \leqslant U_P, \quad \text{where } l = 1 \text{ to } N, \tag{10c}$$

where $x_j(t_i)$ is the maximum controlled displacement in the structure with the jth arrangement of actuators (i.e., jth permutation) at time t_i, h is the total number of time intervals in the time history simulation, q is the total number of possible placement of actuators in the framed structure. x_P, D_p and U_p are, respectively, the maximum permissible displacement, drift ratio and the maximum permissible control force in an actuator. $D_j(t_i)$ is the maximum controlled drift ratio in the framed structure, with jth arrangement of the actuator placement at time t_i. Similarly $U_j^l(t_i)$ is the maximum control force at an actuator location l at time t_i with jth arrangement of actuators.

(b) Peak controlled inter-storey drift ratio

$$\min_p O(N,P) = \min_p \max|D_j(t_i)| \quad \text{where } 1 \leqslant j \leqslant q \quad \text{and} \quad 1 \leqslant i \leqslant h \tag{11}$$

subjected to the constraints given in Eq. (10) where $D_j(t_i) = \max|(x_k(t_i) - x_{k-1}(t_i))|/h_k$ where $1 \leqslant k \leqslant \mathrm{NF}$ and h_k is the height of the $(k-1)$th and kth floors.

(c) Absolute acceleration

$$\min_p O(N,P) = \min_p \max|(a_j(t_i) + a_g(t_i)| \quad \text{where} \quad 1 \leqslant j \leqslant q \quad \text{and} \quad 1 \leqslant i \leqslant h \tag{12}$$

subjected to the constraints given in Eq. (10).
$a_j(t_i)$ is the maximum controlled acceleration in the framed structure with the jth arrangement of actuators (i.e., jth permutation) at time t_i, $a_g(t_i)$ is the ground acceleration at time t_i.

(d) Average control force

$$\min_p O(N,P) = \min_p \max|U_j^{\mathrm{avg}}|, \quad \text{where } 1 \leqslant j \leqslant q, \tag{13}$$

where

$$U_j^{\mathrm{avg}} = \frac{1}{h\,\Delta t}\left[\sum_{i=1}^{h}\left(\sum_{k=1}^{\mathrm{NLOC}} U_k(i)U_k(i)\right)^{1/2} \Delta t\right],$$

where NLOC is the number of locations on the framed structure where control force is employed during active control and Δt is the time step length.

4. MSGNS algorithm

Meta-heuristic algorithms like SA, TS and evolutionary algorithms (EA) (more specifically GA) are popularly used to solve hard combinatorial optimisation problems. Since the cost of each function evaluation in the present combinatorial optimisation problem is high, it is desirable to focus on computational performance of the meta-heuristic algorithm being employed. It is appropriate to choose an algorithm which provides optimal solution with least number of function evaluations. EAs are population-based algorithms and require large number of function evaluations to converge especially, if the domain specific evolutionary operators are not employed. Similarly, parameter settings (cooling schedule) in SA are difficult and are proved to be application specific. Hence, SA either result in generating suboptimal solutions if rather relaxed cooling schedule is set or it becomes computationally tedious, if a stringent cooling schedule is prescribed. Keeping these things in view, a new algorithm called MSGNS algorithm is proposed which makes use of the best features of SA and other guided search algorithm called TS, in order to maintain excellent balance of intensification as well as diversification mechanism. Since we makes use of both SA and TS algorithms as basic search algorithms in the proposed MSGNS algorithm, they are briefly discussed first before presenting MSGNS algorithm.

4.1. Tabu search

TS is a heuristic search technique introduced by Glover [25] and is being used in wide variety of applications. The TS technique uses short-term memory to avoid recycling. Recycling refers to the process of obtaining the same solution repeatedly several times. This recycling is quite common in neighbourhood search algorithms, which generates solutions in a greedy fashion. Recycling ultimately leads the optimisation algorithm to converge to a local optimum. To circumvent this situation and also to cover a wider solution space, TS technique uses short-term memory to mark the recently visited best solutions through neighbourhood search techniques that cannot be accepted as the best solution for a certain number of iterations. These marked solution options are known as '*tabu active*' and the number of iterations for which the move remains *tabu active* is known as '*tabu tenure*'. '*Tabu tenure*' is usually set as $n^{1/2}$ [26], where n is the length of the string representing the solution. In the present work, for a string length of 30 (for 10-storey three bay problem considered as numerical example) '*tabu tenure*' is taken as 5. The *aspiration criterion*, commonly used in TS, is also built into the algorithm. The *aspiration criterion* overrules the '*tabu active*' status, if the set criterion is satisfied. In the optimal placement algorithm, the *aspiration criterion* requires that the solution being considered is superior to the best-ever solution rather than the best among the recently visited solutions.

4.2. Simulated annealing

SA is an iterative search method inspired by the annealing of metals [27,28]. Starting with an initial solution and armed with adequate perturbation and evaluation functions, the algorithm performs a stochastic partial search of the state space. Uphill moves are occasionally accepted with a probability controlled by a parameter called temperature (T). The probability of acceptance of uphill moves decreases as T decreases. At high temperature, the search is almost random, while at low temperature the search becomes almost greedy. At near zero temperature, the search becomes totally greedy, i.e., only good moves are accepted [27,28]. The core of SA algorithm is the Metropolis procedure [29], which simulates the annealing process at a given temperature T. The Metropolis procedure receives as input, the current temperature T, and the current solution *Cur_S* which it improves through neighbourhood search. *Metropolis* must also be provided with the value M, which is the amount of time for which annealing must be applied at temperature T. The procedure *Simulated_annealing* simply invokes *Metropolis* at decreasing temperatures. Temperature is initialised to a value T_0 at the beginning of the procedure, and is slowly reduced (in a geometric or arithmetic progression).

The annealing procedure gets terminated when temperature, T is reduced to a very small value say, 0.001. Eventhough SA has been used extensively for solving combinatorial problems, there are certain problems associated with setting of the cooling schedule of the algorithm, which consists setting up of initial temperature, terminating temperature, temperature decrement, number of Metropolis iterations at each temperature and finally the convergence criteria.

At present, there are no known methods to calculate the cooling schedule for a large range of problems and the values are often set using empirical evidence from experimental runs of the algorithm. This often leads to large number of iterations and function evaluations. In order to overcome this problem, a multiple start algorithm is proposed for the combinatorial optimisation problem of optimal positioning of actuators, which still uses the Metropolis algorithm of SA as a back bone.

In the proposed algorithm, the guided search procedure developed by synthesising SA and TS, is made to restart from the current best solution after completing specified number of iterations, instead of performing the metropolis iterations repeatedly on the probabilistically accepted solution until termination. The algorithm terminates after K restarts without improvement. The MSGNS algorithm is shown in Figs. 1–5.

One of the important issues in designing the hybrid meta-heuristic algorithm is to improve the diversification mechanism in order to smartly search the solution space with least number of function evaluations. For example a simple greedy search algorithm, systematically explores the neighbourhood of a seed solution, and discards infeasible/inferior solutions immediately after solution evaluation while it keeps better solutions as new seed solutions for repeating the search procedure. The procedure soon reaches a dead end when no more better solutions can be found within a seed solution's neighbourhood. This shortsighted strategy inevitably causes local searches to be trapped in local optima very quickly. TS algorithm can overcome this situation, and help to move out of the dead ends of these greedy searches. TS, in addition to systematically exploring the neighbourhood, intelligently goes a step further to label along its exploring trail and to accept unexploited local optima irrespective of their quality when compared to seed solutions. In contrast SA obtains a neighbourhood solution randomly, but accepts it with a probability reflecting annealing property. Therefore, both TS and SA diversify the search in order to escape from local optima in contrasting styles. While TS works in a deterministic way so that its exploring trail is fixed, SA does so in a non-deterministic way and its exploring trails vary in different tries. Keeping these things in view, the SA features have been synthesised with TS in order to enhance the diversification mechanism in the proposed algorithm.

```
Algorithm Multi_Start_GNS
Configure parameters K, L, M, N, α and T0, NS
Set Tabu_List empty
Sol_good = Sol_random;
Best_ever_sol = Sol_good;
Non_improving = 0;
WHILE (Non_improving < K)
DO / * k-restart */
    Sol = Sol_good
    Sol_good_new = Tabu_GNS ( Sol, L, M, N, α, T0, NS, Best_ever_sol )
    IF  FITNESS(Sol_good__new) < FITNESS (Sol_good) THEN
        Sol_good = Sol_good__new;  /*update current best solution*/
        Best_ever_sol = Sol_good;
        Non_improving = 0
    ELSE
        Non_improving = Non_improving + 1;
    END IF
OUTPUT Sol_good;
```

Fig. 1. *Multi_Start_GNS* algorithm.

```
Algorithm Tabu_GNS (Sol, L, M, N, α, T0, NS, Best_ever_sol)
Sol_best = Sol;
Non_improving = 0;
WHILE ( Non_improving < L)
DO
   Sol_new = Tabu_Metropolis (Sol, M, α , T0, NS, Best_ever_sol)
                                                  /* Perform Metropolis iterations */
   Sol_local = Tabu_NH_search(Sol_new, N, NS, Best_ever_sol);
                                                  /*obtain a non-Tabu neighbour */
   IF FITNESS(Sol_local) <FITNESS (Sol_best) THEN
      Sol_best = Sol_local;
      Non_improving = 0;
    IF (FITNESS(Sol_best) < FITNESS(Best_ever_sol)) Best_ever_sol=Sol_best;
   ELSE
      Non_improving = Non_improving + 1;
      Sol  = Sol_local;
   END IF
ENDDO
RETURN Sol_best;
```

Fig. 2. *Tabu_GNS* algorithm.

Another important desirable feature of the meta-heuristic algorithm is the intensification mechanism. Both SA and TS does not deal this issue specially. In most of the algorithms this issue of intensification is dealt as post-processing step by identifying the best-ever solution. It means that intensification is dealt separately as a post-processing step rather than embedding in the algorithm. The MSGNS algorithm precisely incorporates the intensification mechanism into the algorithm. The MSGNS guides the TS algorithm to search around the current best solution. If a new better solution is found within the specified number of K restarts of TS, the current best solution is updated and a new series of restarts are initiated. Otherwise, if K consecutive restarts are finished without finding better solution, this procedure stops. Therefore, the intensification, which is in favour of the current best solution, always plays its role during the search process.

4.3. Neighbourhood search algorithms

During designing neighbourhood algorithms, several important issues need to be taken into consideration. The neighbourhood search mechanisms, which usually have a strong connection with the problem and solution representation, provide spaces within which the searches are conducted. For the optimal positioning problem, the solution string is represented by 0 or 1 which indicate whether an actuator is present in a particular bay or not. The length of the string is therefore equal to the total number of bays ($S \times n_b$) in the framed structure. While '1' indicates the presence of an actuator in a bay, '0' indicates that there is no actuator. For the optimal placement problem, we proposed the following four neighbourhood search algorithms. They are *invert, permutation, swap* and finally *smart_swap*.

Invert works in two phases. In the first phase, it randomly picks a location in a string and transforms the value to '1' if '0' is present and vice versa. In order to maintain the balance related to desired number of actuators, in the second phase, it inverts the randomly located '0' to '1' or '1' to '0' depends upon the first

```
Algorithm Tabu_Metropolis (Sol_new, M, α , T0, NS, Best_ever_sol)

ITER = 0;

Sol = Sol_new;

WHILE ( ITER <  M)

DO

    Sol_local = = Neighbourhood_search (Sol, NS, Best_ever_sol)

    DELTA = FITNESS (Sol_local) – FITNESS (Sol);

    IF (DELTA≤0) THEN Prob = 1

    ELSE Prob = e^(-DELTA/T)

    ENDIF

    IF random (0,1) ≤ Prob THEN

    Sol =  Sol_local;

IF(FITNESS (Sol_local) < FITNESS (Best_ever_sol)) Best_ever_sol=Sol_local;

  Record Sol_local into Tabu_list

    ENDIF

    ITER = ITER + 1;

 ENDDO

 Update annealing Temperature T = α*To;

 RETURN Sol;
```

Fig. 3. *Tabu_Metropolis* algorithm.

phase operation. The location chosen randomly in the first phase will be masked, so that it will not be considered again in the second phase. This will prevent undoing of the changes made in the first phase and at the same time maintain the balance related to the desired number of actuators.

Permutation is another neighbourhood search algorithm built into the proposed multiple start algorithm, which chooses two random locations in the string and the entire string within the chosen two random positions is inverted (reversed).

The third neighbourhood search algorithm is *swap*, which is less disruptive when compared to *permutation*. Similar to *permutation*, swap also randomly chooses two random locations in the string and swaps their respective values. While implementing this algorithm, the first location is chosen randomly and the value at the random location is recorded, i.e., '0' or '1'. Based on this information, the second random location is chosen such that it should have a different value than the first location. After, confirming these locations, the swap operation will be executed.

The *smart_swap* essentially works similar to *swap*. However, the smart swap acts according to the fitness-function. In the present algorithm, the maximum storey-wise fitness values (of desired objective function) is first obtained and are assigned as fitness value to the corresponding group of the given string. *Smart_swap* makes use of this information to perform *swap*. The storey of the frame, which has an actuator and has a minimum fitness value, is chosen and its actuator position is moved to a storey whose fitness value is high. Alternatively, the smart swap also places the actuator randomly by removing from the storey with minimum fitness value. We call the first one which places the actuator deterministically as *smart_swap1* and second one which places the actuator randomly as *smart_swap2*.

```
Algorithm Tabu_NH_search (Sol, N, NS, Best_ever_sol)
Sol_best =  Sol;
Non_improving = 0;
WHILE ( Non_improving < N)
DO
    Sol = Sol_best;
     Sol_local = Neighbourhood_search (Sol, NS, Best_ever_sol);
                              /*obtain a non-Tabu neighbourhood sol. */
    IF FITNESS(Sol_local) <FITNESS(Sol_best) THEN
         Sol_best = Sol_local
IF(FITNESS(Sol_local) < FITNESS(Best_ever_sol)) Best_ever_sol=Sol_local;
         Non_improving = 0;
         Record sol_local in Tabulist;
    ELSE
         Non_improving = Non_improving + 1;
   END IF
ENDDO
RETURN Sol_best;
```

Fig. 4. *Tabu_NH_Search* algorithm.

```
Algorithm Neighbourhood_Search(Sol_p, NS, Best_ever_sol)
Sol = Sol_p;
Set TS =1;      / * Set Tabu search  status as inactive */
Set ASPIRATION = 0; / * set Aspiration criteria as negative */
 WHILE( TS = 1 .and.  ASPIRATION = 0)
 DO
Sol_local = Sol;
NS = Random(1, p);
                / * p is the number of neighbourhood search algorithms available */
IF (NS = 1)sol_new =  invert(sol_local);
IF (NS = 2)sol_new =  Permutation(sol_local);
IF (NS = 3)sol_new =  Swap(sol_local);
IF (NS = 4)sol_new =  Smart_swap1(sol_local);
IF (NS = 5)sol_new =  Smart_swap2(sol_local);
   TS = 0;
IF ( Sol_new . is. TABU ACTIVE) THEN
   TS =1;
   IF( FITNESS(Sol_new) < FITNESS( Best_ever_sol)) ASPIRATION = 1;
 ENDIF
IF(TS=0 .or. ASPIRATION = 1) Sol = sol_local;
ENDDO
RETURN( Sol);
```

Fig. 5. Neighbourhood search algorithm.

5. Implementation details of MSGNS algorithm

The MSGNS algorithm essentially consists of the following six different modules. They are: *Multi_start_GNS*, *Tabu_GNS*, *Tabu_Metropolis*, *Tabu_NH_search*, *Neighbourhood search* and *Fitness* function.

Various components of *Multi-start_GNS* algorithm are presented in Fig. 1. It generates the initial solution randomly or using some previous knowledge about the problem. It evaluates the fitness of the initial solution using given fitness and records it as the best-ever and passes the string to the *Tabu_GNS* module along with its associated fitness. This module sets the number of iterations K, for *Multi-start_GNS* algorithm, L for *Tabu_GNS* and M for *Tabu_Metropolis* and N for *Tabu_NH_search*. Apart from this, the initial temperature T_0, the type and magnitude of temperature decrement operator α is also set in this module. The *Multi-start_GNS* terminates when there is no improvement in the fitness values for last consecutive K iterations of *Tabu_GNS*.

The *Tabu_GNS* module given in Fig. 2 is built with both TS and Metropolis algorithmic features in order to improve the diversification mechanism of the proposed meta-heuristic algorithm. *Tabu_GNS* works basically as a coordinator to the *Tabu_Metropolis* and *Tabu_NH_search* algorithms. The input string (represented solution) obtained from *Multi-start_GNS* is first passed to *Tabu_Metropolis* and the new string obtained after given number (say M) of metropolis iterations will be passed to the *Tabu_NH_search* algorithm to perform the set number of neighbourhood searches using the application specific neighbourhood algorithms devised for the optimal placement problem. The *Tabu_GNS* algorithm terminates and returns the improved string, obtained by calling alternatively the *Tabu_Metropolis* and *Tabu_NH_search*, when there is no improvement in the solution for L consecutive moves.

Tabu_Metropolis is basically the metropolis algorithm which reflects the annealing property of the algorithm. The algorithm is shown in Fig. 3. It uses the metropolis criteria to decide whether to accept an inferior solution or not. M number of metropolis iterations is performed to reflect the stabilisation criteria (at each temperature) of the annealing process. Apart from this, the *Tabu_Metropolis* is also built with TS features to prevent recycling. The '*tabu tenure*' is set to five and the tabu list will be updated continuously whenever a new move based on neighbourhood search is generated. The new moves obviously replace the oldest move in the tabu list so that the recently visited moves are preserved. The '*aspiration criterion'* is also built into the proposed algorithm which overrules the tabu, when the solution obtained is superior to the best-ever solution.

Tabu_NH_search given in Fig. 4 takes the input string from the *Tabu_GNS* algorithm and performs neighbourhood search repeatedly making use of the neighbourhood search algorithms discussed in the earlier sections. The *Tabu_NH_search* algorithm basically works similar to a greedy search algorithm. Whenever a string with better fitness value is obtained during neighbourhood search, the current solution is updated with the better solution and proceeds with the neighbourhood search. The *Tabu_NH_search* algorithm terminates when there is no improvement in the last N neighbourhood searches.

Finally, the *Neighbourhood_search* algorithm given in Fig. 5 performs the neighbourhood search using any one of the neighbourhood search algorithms discussed in the earlier section. However, in the present work, we prefer to use all the neighbourhood search algorithms together in a single problem, as we have some evidence based on some test results that the present strategy works better rather than using a single algorithm at a time. Whenever, a *Neighbourhood_search* algorithm is called either by *Tabu_Metrpolis* or *Tabu_NH_search* algorithms, one of the five neighbourhood algorithms is randomly picked and used for generating the neighbourhood solution. The Neighbourhood search algorithm is built with TS features, and hence it prevents the generated neighbourhood solutions that are found to be '*tabu active*'. However, if '*aspiration Criterion*' is satisfied, the '*tabu active*' status will be overruled and the solution will be accepted.

6. Performance enhancement techniques

Since detailed finite element models are employed for structural analysis in the present work, the function evaluation (i.e., function FITNESS in the algorithms given in Fig. 1–5) is computationally expensive especially for large size problems. In view of this, two performance enhancement techniques have been implemented in the present work. One is called *cache-fetch*, which fetches the solution from the already solved string patterns, in case the pattern repeats after several cycles. The second one is the *Riccati-fetch*, which fetches the Riccati solution from already solved string patterns, if there is a match. It may be noted that Riccati solution takes substantial amount of computational time in the function evaluation. It is also appropriate to mention here that while *cache-fetch* can fetch the values from the patterns already visited while solving the problem with a particular earthquake acceleration data, the *Riccati-fetch* can fetch value from the matching patterns generated even while solving the problem with other earthquake acceleration data.

The string patterns and their respective fitness values and Riccati solution are stored in the secondary storage using a binary tree (BT) data structure. The BT data structure enables to store the data in an efficient manner and retrieve back without laborious search.

7. Parameter settings for MSGNS algorithm

The performance of the proposed MSGNS algorithm depends on the optimal setting of parameters like K and L. These two parameters are the key factors, which control and balance the diversification and intensification mechanisms built into the proposed algorithm. Hence, it is desirable to set larger values for both K and L. But larger K and L values lead to higher computational cost. In view of this, these parameters need to be optimised for improved computational performance.

In order to arrive at optimal parameters for MSGNS algorithm, some small test problems have been solved. Based on these parametric studies, the values for K and L are set as 5 and 4, respectively. Similarly M and N are taken as 8 and 5, respectively for all the numerical simulation studies carried out in this paper. The initial temperature T_0 is taken as 100 and the temperature decrement factor, α is taken as 0.95. The details of these parametric studies are deliberately omitted in this paper as they are of least significance.

8. Numerical studies

A 10-storey three-bay framed structure [30] is considered as a numerical example to demonstrate the effectiveness of the proposed MSGNS algorithm for optimal placement of actuators. The framed structure and the arrangement of a typical active tendon is shown in Fig. 6. For multiple bay structural frames, more general and sophisticated structural modelling methods such as finite element method are desirable. In the present work, all test cases are modelled using a two-node, six degrees-of-freedom planar beam element.

Fourteen different earthquake records as given in Table 1 have been used in the present case study. These earthquake records with different strengths, duration and other characteristics are employed as the external excitations to the 10-storey framed structure with ATM to observe the structural responses and to determine the appropriate actuator placements based on the chosen optimisation criterion.

Further, numerical studies have been carried out by considering various optimisation criteria and also for variable number of actuators, using all the earthquake records given in Table 1. However, as these studies are exhaustive, only some selected studies are presented in this paper.

8.1. Optimisation criterion 1: controlled peak displacement

Peak controlled displacement is considered as the first optimisation criterion. Numerical simulation studies have been carried out by varying the number of actuators from 2 to 10 and arriving at the optimal location using the proposed MSGNS algorithm for all the earthquake records given in Table 1. The peak controlled displacements are obtained with actuators placed at optimal locations as given by the MSGNS algorithm. These optimal controlled displacements are compared with two alternative cases where the actuators are placed either on top or bottom floors of the building frame. These top and bottom floor positions are considered for comparison purposes as these happen to be the most probable. Fig. 7 shows the results obtained for 4, 6, 8 and 10 actuators. A close look at the figures indicates that the performance is superior with optimally located actuators in the building frame when compared to arbitrary (of course with some intuition) placement of actuators, i.e., either on top or bottom floors. It can also be observed that the controlled maximum displacements decrease with the increase in number of actuators, which is quite obvious. Further work is however needed to identify the optimal number of actuators for a given building frame and earthquake intensity.

There are several parameters to be considered in an earthquake record: amplitude, number of peak accelerations, duration and predominant period, etc. These parameters differ in each earthquake record. However, for the example building frame considered in this paper, it has been observed that the optimal

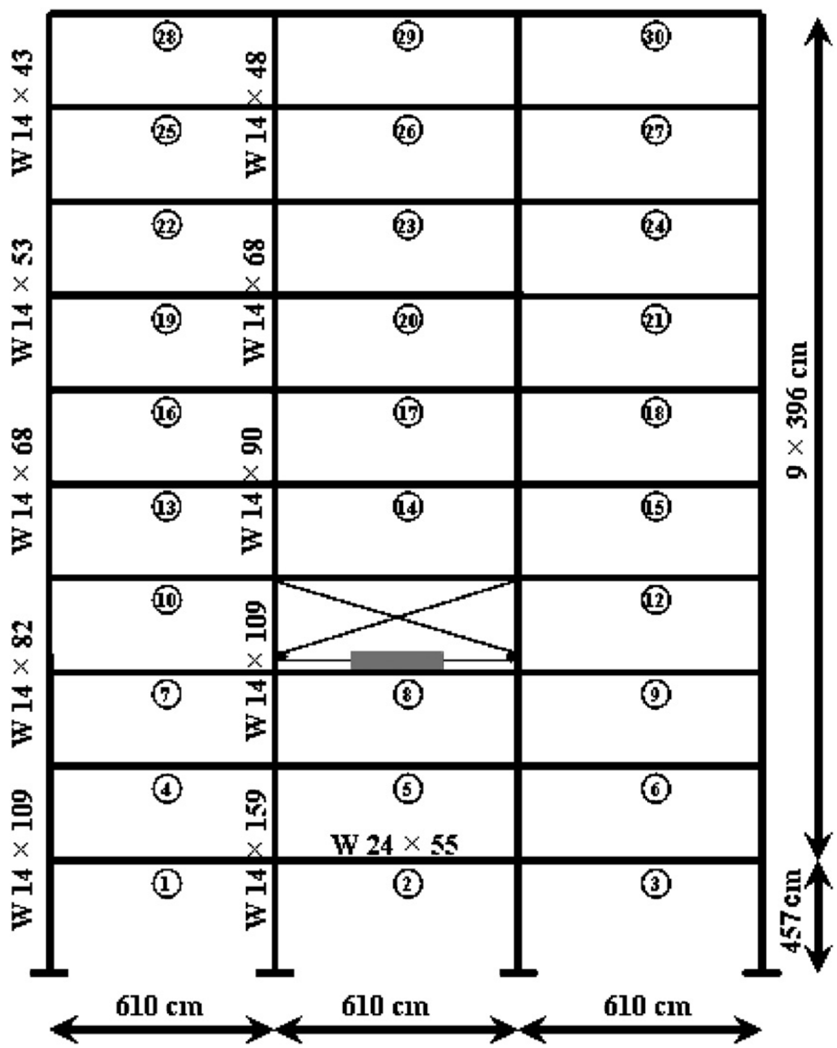

Fig. 6. Ten-storey framed structure and a typical tendon arrangement at location 11.

Table 1
Details of earthquake records

S. no.	Earthquake name	Location	Peak accln. ($\times g$)	Year
1	ElCentro	California, USA	0.3495	18.05.1940
2	Hachinohe	Japan	0.2294	16.05.1968
3	Northridge	California, USA	0.8428	17.01.1994
4	Kobe	Japan	0.8337	17.01.1995
5	Bhuj	India	0.1000	26.01.2001
6	San Fernando	California, USA	1.0753	09.02.1971
7	Cape Mendocino	Caifornia, USA	1.4967	25.04.1992
8	Big Bear	California, USA	0.2252	28.06.1992
9	LomaPrieta	California, USA	0.6297	17.10.1989
10	Chile	Central Chile	0.7207	03.03.1985
11	Chi Chi	China	1.1533	20.09.1999
12	Tabas	Iran	0.8518	16.09.1978
13	Nahanni	Canada	2.0865	23.12.1985
14	Duzce	Turkey	0.5137	11.12.1999

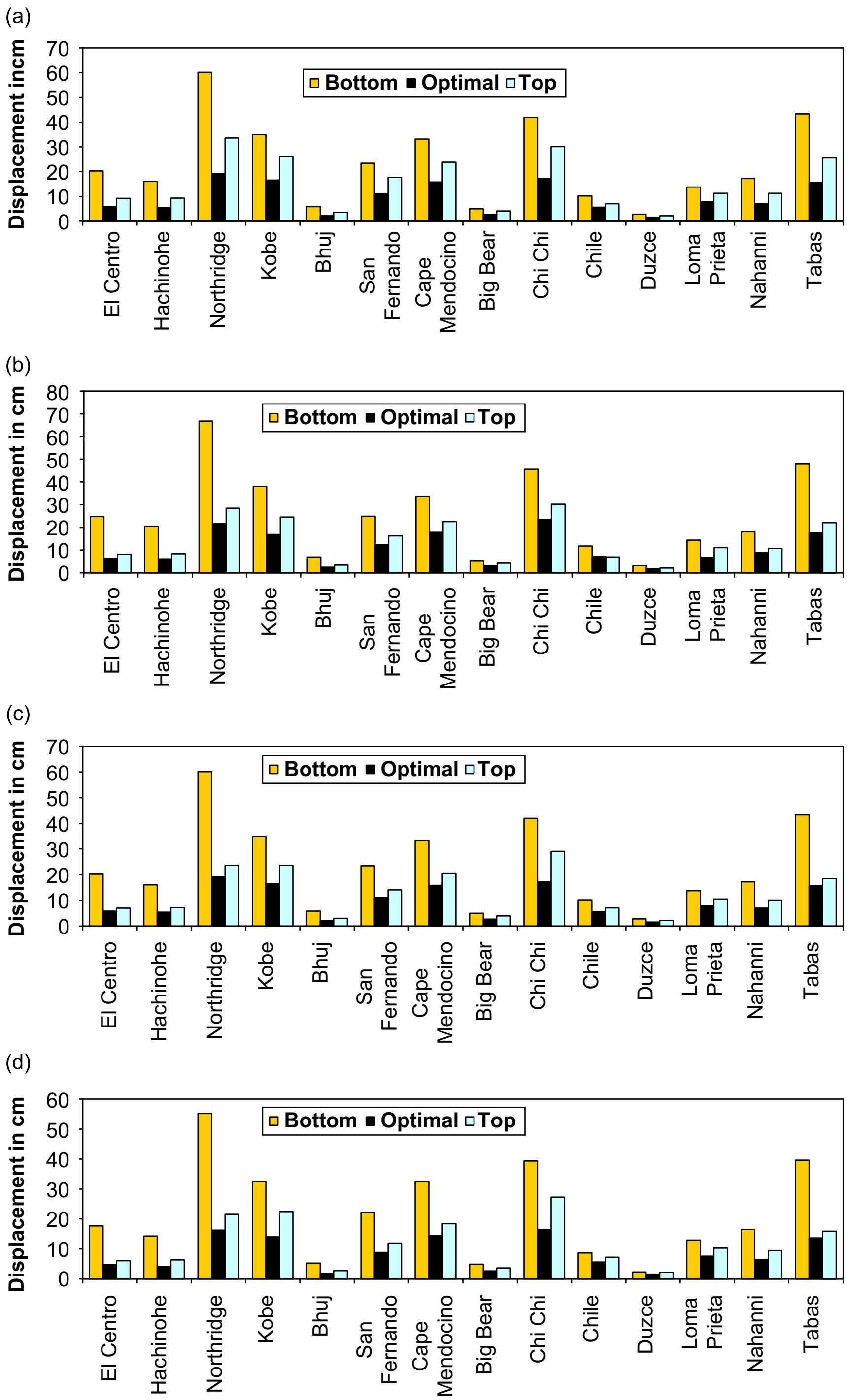

Fig. 7. Performance of control system with optimally placed actuators (based on displacement criterion) using MSGNS algorithm. (a) Four actuator problem, (b) six actuator problem, (c) eight actuator problem and (d) ten actuator problem.

actuator locations are same for all the earthquake records considered. Hence, the characteristics of the earthquake are not the influencing factors for optimal actuator locations in tall buildings when the optimisation criterion is considered as peak controlled displacement. The optimal locations for 2, 4, 6, 8 and 10 numbers of actuators are presented in Table 2. Each location number given in the table indicates the respective bay in the frame. The bays of the framed structure shown in Fig. 6 are numbered sequentially as 1–30 from bottom to top floors starting from left to right as indicated in the figure. It can be observed from the results shown in Table 2 that the optimisation criterion based on peak controlled displacement clearly favours the top few floors for actuator placement. Table 2 also shows the details related to actuator locations for arbitrary placement either on top or bottom floors of the 10-storey building frame for which the controlled displacements for various earthquakes are evaluated for comparison purposes and given in Fig. 7.

This study clearly demonstrates that the control performance of the actively controlled framed structure can be maximised by judicious placement of the given number of actuators rather than placing them arbitrarily either on top or bottom floors of the framed structure. Hence, it is rather essential to use a sophisticated combinatorial optimisation algorithm like the proposed MSGNS algorithm to arrive at optimal locations for the given number of actuators in order to maximise the control performance.

8.2. Optimisation criterion 2: controlled peak drift ratio

In order to verify the influence of the earthquake excitation on the optimal placement problem, the optimisation criteria has been changed and chosen as peak controlled drift ratio instead of peak displacement and numerical studies are carried out using the proposed MSGNS algorithm. The framed structure shown in Fig. 6 is taken as numerical example and has been solved for optimal placement using six actuators and employing selective earthquake records given in Table 1. In contrast to the earlier observations, different optimal locations (for peak drift ratio as optimisation criterion) have been obtained for different earthquake data. The details of optimal locations for various earthquake excitation records are presented in Table 3. A close look at the table indicates that the earthquake parameters certainly influence the optimal location when drift ratio is considered as an optimisation criterion. It can also be observed that the optimal actuator location patterns are entirely different from the optimisation criteria with peak controlled displacement. Further, the actuators are fairly distributed in the entire frame. This is in contrast to the earlier observation while peak displacement is considered as optimisation criterion, where the optimal locations of actuators are biased clearly towards top floors.

The maximum controlled displacements and drift ratios of 10-storey framed structure with ten actuators placed at optimal locations as given by the proposed MSGNS algorithm for certain selective earthquakes listed in Table 1 are compiled. These optimal controlled displacements are compared with three alternative

Table 2
Optimal actuator locations for the 10-storey framed structure obtained using MSGNS algorithm with peak controlled displacement as optimisation criteria

S. no	Number of actuators	Criteria	Optimal actuator locations
1	2	Arbitrary (Top)	29, 30
		Optimal	29, 30
		Arbitrary (Bottom)	2, 3
2	4	Arbitrary (Top)	27, 28, 29, 30
		Optimal	19, 20, 29, 30
		Arbitrary (Bottom)	1, 2, 3, 6
3	6	Arbitrary (Top)	25, 26, 27, 28, 29, 30
		Optimal	17, 20, 23, 26, 29, 30
		Arbitrary (Bottom)	1, 2, 3, 4, 5, 6
4	8	Arbitrary (Top)	23, 24, 25, 26, 27, 28, 29, 30
		Optimal	11, 14, 17, 20, 23, 26, 29, 30
		Arbitrary (Bottom)	1, 2, 3, 4, 5, 6, 8, 9
5	10	Arbitrary (Top)	21, 22, 23, 24, 25, 26, 27, 28, 29, 30
		Optimal	13, 14, 15, 17, 20, 23, 25, 26, 28, 29
		Arbitrary (Bottom)	1, 2, 3, 4, 5, 6, 8, 9, 11,12

Table 3
Optimal actuator locations for the 10-storey framed structure using design criteria based on drift ratio, abs. acceleration and average control force for various earth quake

S. no.	Earthquake details	Number of actuators	Optimal actuator locations		
			Criterion based on drift ratio	Criterion based on abs. acln.	Criterion based on avg. CF
1	El Centro	6	8,11,15,18, 24, 27	4, 5, 7, 20, 25, 28	1, 3, 7, 12, 28, 30
2	Hachinohe	6	10,11, 13, 22, 26, 30	5, 8, 10, 15, 18, 30	1, 3, 26, 27, 28 30
3	San Fernando	6	7, 10, 18, 21, 26, 30	8, 22, 23, 25, 27, 28	1, 3, 19, 24 25 30
4	Cape Mendocino	6	8, 12, 13, 16, 26, 29	2, 5, 7, 18, 19, 28	1, 3, 25, 27, 28, 30
5	Loma Prieta	6	6, 9, 12, 13, 17, 26	1, 4, 16, 21, 28, 30	1, 3, 25, 27, 28, 30
6	Duzce	6	13, 19, 21, 22, 28, 29	13, 15, 17, 19, 22, 23	1, 3, 19, 21, 28, 30

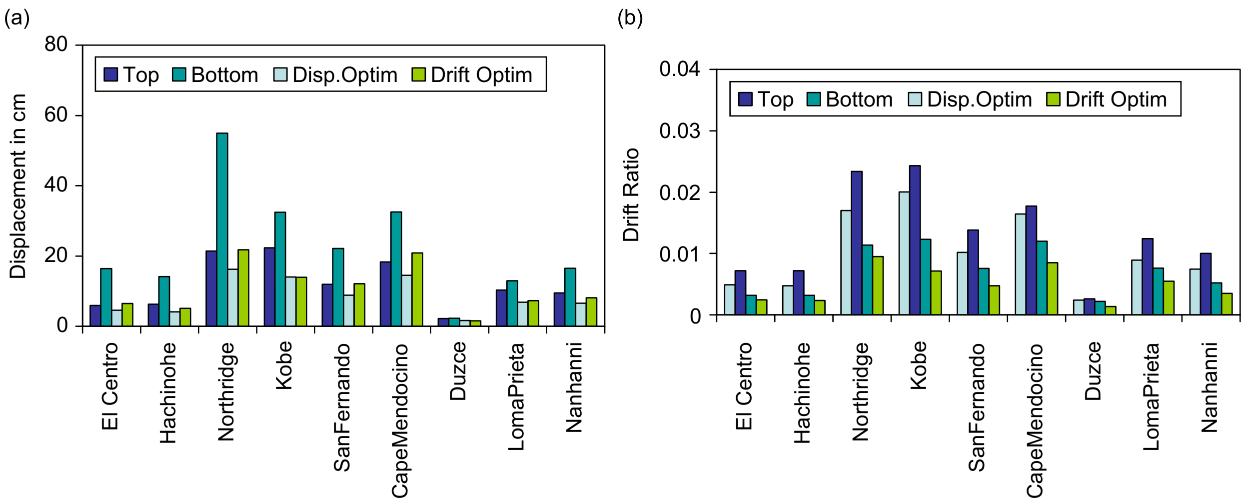

Fig. 8. Performance of control system with optimally placed actuators (ten numbers) with optimisation criteria based on peak controlled displacement and drift ratio using MSGNS algorithm. (a) Peak controlled displacement and (b) peak controlled drift ratio.

cases where the actuators are placed either on top or bottom floors of the building frame or on optimal locations dictated by the displacement optimisation criterion. Fig. 8 shows the peak displacements and peak drift ratios obtained for various selective earthquakes. A close look at Fig. 8 indicates that the peak controlled displacements are minimal for the optimal locations dictated by the displacement criterion. Similarly, the peak controlled drift ratio is minimal for the optimal actuator locations dictated by the drift ratio criterion. This study once again demonstrates the effectiveness of the proposed MSGNS algorithm for optimal positioning of actuators for active control of structures.

Table 4 shows the peak controlled displacements and peak controlled drift ratios for varied number of actuators. Loma Prieta earthquake excitation data is used for this study. The results presented here in Table 4 are obtained for all the alternative test cases, i.e., placing the actuators on top, bottom and also optimally placed based on displacement and drift criterion. A close look at Table 4 indicate that, the peak controlled drift ratio and displacement values reduces with increase in the number of actuators, which is quite obvious. However, the following are some of the interesting observations that can be made on the rate of reduction of these two parameters.

It can be observed that the reduction in peak controlled displacement, when compared to uncontrolled response, is of the order of 27–64% with the increase in the number of actuators from two to ten, when displacement is considered as optimisation criteria. Similarly the reduction in peak controlled displacements

Table 4
Optimal actuator locations based on displacement and drift criteria for three-bay 10-storey framed structure subjected to Loma Prieta earthquake

S. no.	Number of actuators		Optimal actuator locations	Disp (cm)	% reduction over peak disp.	Drift	% reduction over peak drift
1	2	Top	29, 30	10.53	26.98	0.0134	11.84
		Optimal (Disp.)	29, 30	10.53	26.98	0.0117	23.02
		Optimal (Drift)	10, 14	11.75	18.51	0.0109	28.29
		Bottom	2, 3	14.39	0.20	0.0147	3.29
2	4	Top	27, 28, 29, 30	11.17	22.54	0.0128	15.79
		Optimal (Disp.)	19, 20, 29, 30	8.84	38.70	0.0091	40.13
		Optimal (Drift)	9, 12, 15, 22	9.94	31.06	0.0079	48.03
		Bottom	1, 2, 3, 6	14.35	0.49	0.0142	6.58
3	6	Top	25, 26, 27, 28, 29, 30	11.10	23.02	0.0122	19.74
		Optimal (Disp.)	17, 20, 23, 26, 29, 30	6.85	52.50	0.0076	50.00
		Optimal (Drift)	6, 9, 12, 13, 17, 26	7.82	45.77	0.0055	63.82
		Bottom	1, 2, 3, 4, 5, 6	14.33	0.62	0.0140	7.90
4	8	Top	23, 24, 25, 26, 27, 28, 29, 30	10.77	25.31	0.0104	31.58
		Optimal (Disp.)	11, 14, 17, 20, 23, 26, 29, 30	5.69	60.54	0.0058	61.84
		Optimal (Drift)	5, 8, 9, 12, 15, 18, 20, 25	6.34	56.03	0.0045	70.39
		Bottom	1, 2, 3, 4, 5, 6, 8, 9	13.91	3.54	0.0131	13.82
5	10	Top	21, 22, 23, 24, 25, 26, 27, 28, 29, 30	10.34	28.29	0.0091	40.13
		Optimal (Disp.)	13, 14, 15, 17, 20, 23, 25, 26, 28, 29	5.14	64.35	0.0049	67.76
		Optimal (Drift)	5, 6, 7, 8, 9,12, 15, 18, 21,27	6.06	57.98	0.0037	75.66
		Bottom	1, 2, 3, 4, 5, 6, 8, 9, 11,12	13.33	7.56	0.0121	20.39

with drift ratio as optimisation criteria is of the order of 18–58% for varied number of actuators. This clearly indicates that peak displacements can more effectively be controlled by employing the optimisation criteria based on displacement.

Similar observation can be made with respect to drift ratios also. The reduction in the peak controlled drift ratio, when compared to uncontrolled response, is of the order of 23–68% with displacement as optimisation criteria and are of the order of 28–76% with drift ratio as the optimisation criteria. This clearly re-emphasis the fact that the chosen controlled parameter can be effectively optimised by choosing the same parameter as the objective function.

Further, Table 4 also presents the controlled displacement and drift ratios obtained when the actuators are placed arbitrarily either on top floors or bottom floors of the 10-storey, three-bay building frame. The locations of the actuator placements are clearly indicated in the table. It can be observed from the results presented in Table 4 that the control performance, i.e., reduction in peak displacement and drift ratios is significantly lower with arbitrary placement of actuators either on top or bottom floors of the building frame, when compared to the performance with optimal placement of actuators with the proposed MSGNS algorithm. For example, with ten actuators placed on the bays of top floors of the framed structure, the displacement and drift ratios are reduced only by 29% and 40%, respectively. Similarly, for the arrangement of actuators on bottom floors of the building frame, the reduction in controlled displacement and drift ratios is 8% and 20%, respectively. This clearly indicates that arbitrary placement of actuators results in substantial loss of control performance. Hence, it is extremely important to use an advanced combinatorial optimisation algorithm for optimal placement in order to maximise the performance. Further, it can also be observed that, among the two cases of arbitrary placement of actuators (i.e., on top or bottom floors), placement of actuators on top floors gives comparatively better control performance than the arrangement of actuators at the bottom floors of the building frame.

8.3. *Optimisation criterion 3: controlled peak absolute acceleration*

Peak absolute acceleration is considered as the third optimisation criterion. Table 3 shows the optimal actuator locations for some selective earthquake acceleration records given in Table 1. Only six actuators are considered in this study. It can be observed that similar to the earlier optimisation criterion based on drift ratios, the optimal locations of actuators is sensitive to the earthquake characteristics, if one intends to optimise the absolute accelerations.

8.4. *Optimisation criterion 4: average control force*

Control force is another design criterion which influences the active control of structures. Hence, average control force is considered as the fourth optimisation criterion. Table 3 shows the optimal actuator locations for various earthquake acceleration data using six actuators. This study also establishes that the earthquake characteristics can influence the optimal placement, if average control force is considered as a design criterion.

Fig. 9 show the peak control parameters like displacement, drift ratio, absolute acceleration and average control force for the 10-storey framed structure with six optimally placed actuators, using all the four optimisation criteria considered in this paper. The results are presented in the figures only for some selective earthquake acceleration records given, in order to maintain better clarity. The following are some of the observations based on the results furnished in Fig. 9:

(i) The chosen optimisation criterion obviously influences the respective parameter to behave better when compared to other parameters in the controlled system.
(ii) Optimisation of average control force leads to increase in peak displacement and drift ratios. However, they are within the constraints stipulated for the problem.

We can observe from Fig. 9, that the variation in peak controlled absolute acceleration is not very significant for all the four optimisation criteria considered in this paper. Hence, it may be sufficient if we consider only controlled displacement, drift ratio and control force as optimisation criteria and it will automatically (at least reasonably) satisfy the obvious fourth criterion, i.e., absolute acceleration. This gives a lead to further research in this area. One can alternatively choose to simultaneously optimise controlled parameters like displacement, drift ratio and control force using multi-criteria optimisation techniques without loosing much leverage on reduction of absolute acceleration.

8.5. *Optimal actuator placement using multiple optimisation criteria*

As mentioned earlier, the most effective way of solving the optimal actuator placement problem is to solve as a multi-criteria optimisation problem. Here an attempt has been made to arrive at optimal actuator positions by aggregating the multiple objectives with weighting factors. Based on the reasoning given in the earlier section, the objective function is formulated by combining only the three objective functions related to peak controlled displacement, drift ratio and average control force. The combined objective function can be

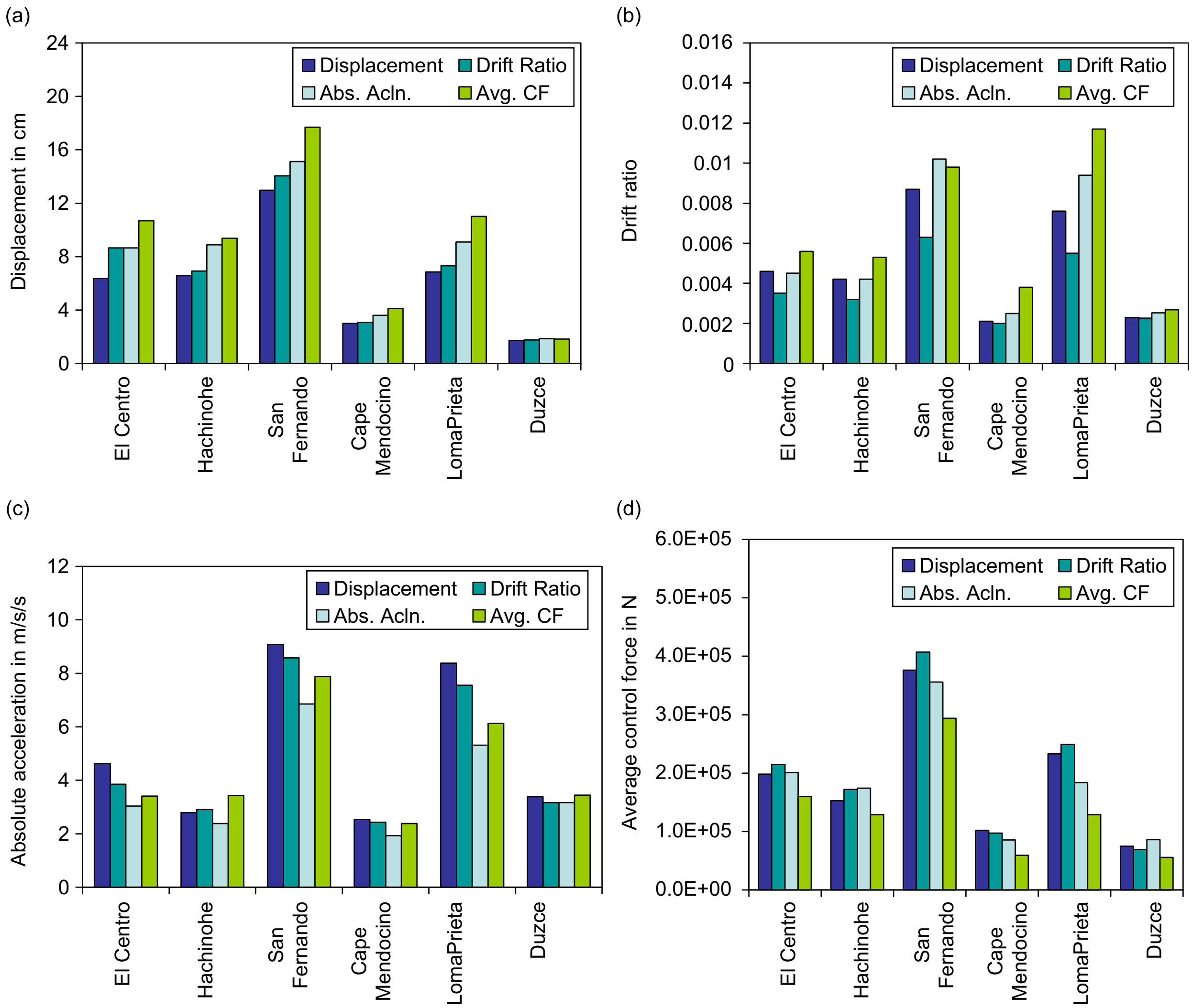

Fig. 9. Peak controlled response of 10-storey framed structure with six optimally placed actuators using all the four optimisation criteria. (a) Peak controlled displacement, (b) peak drift ratio, (c) peak absolute acceleration and (d) average control force.

written as

$$o(p) = \alpha\sigma_f + \beta\sigma_d + \gamma\sigma_{cf}, \tag{14}$$

where σ_f is the peak controlled displacement normalised with uncontrolled displacement, σ_d is the peak controlled drift ratio normalised with uncontrolled drift ratio and finally σ_{cf} is the average control force normalised with the actuator capacity. α, β and γ are the respective weightages with $\alpha + \beta + \gamma = 1$. By varying α, β and γ one can obtain several non-dominated solutions (Pareto optimal) for actuator placement. Some selective results of the optimal solutions obtained using MSGNS algorithm using the weighted optimisation criteria given in Eq. (14) are shown in Table 5. A close look at Table 5 indicate that the solutions obtained using the weighted aggregating approach are a set of non-dominated solutions and the user has a wider choice to choose one among them through qualitative assessment based on the site-specific requirements like actuator capacity, importance of the structure, dynamic characteristic history of the earthquakes at the site and finally financial considerations. The Pareto optimal curve obtained using the multi-criteria optimisation with three objectives is shown in Fig. 10.

8.6. Sixteen-storey building frame

In order to emphasise further, the significance of alternative optimisation criteria and their influence in optimal placement of actuators, a single bay 16-storey building framed structure solved by Li et al. [17] is taken as second numerical example. El-Centro earthquake data is considered for arriving at optimal placement of varied number of actuators. The structural parameters of the 16-storey building example are given in Table 6. Since these structural parameters as given by Li et al. [17] are only for shear building model, we have considered only shear building model for this numerical example. It is however, appropriate to point out here that the numerical example considered has only single bay. Hence, the question of optimal placement with respect to a bay in each floor does not arise. In view of this, use of detailed finite element analysis may not have a significant influence on the positioning of the actuators for this numerical example. The optimal positioning of actuators using all the four optimisation criteria is shown in Table 7. The optimal locations given in the table are in the form of bay numbers, which are numbered from bottom floor to top floor. These

Table 5
Pareto optimal solutions for optimal actuator placement

S. no.	Locations	α	β	γ	Displacement (mm)	Drift ratio	Average CF (kN)
1	17, 20, 23, 26, 29, 30	1.00	0.00	0.00	63.60	0.0046	197.62
2	8, 11, 15, 18, 24, 27	0.00	1.00	0.00	86.44	0.0035	214.78
3	1, 3, 7, 12, 28, 30	0.00	0.00	1.00	106.80	0.0056	160.13
4	15, 19, 22, 27, 28, 29	0.50	0.00	0.50	93.82	0.0051	180.06
5	4, 13, 14, 15, 28, 29	0.50	0.50	0.00	98.12	0.0041	212.45
6	13, 14, 19, 22, 26, 28	0.40	0.40	0.20	102.29	0.0044	197.46
7	8, 11, 16, 17, 27, 28	0.40	0.50	0.10	107.27	0.0043	204.11
8	14, 18, 19, 24, 27, 28	0.40	0.15	0.45	104.35	0.0049	184.99
9	4, 6, 10, 27, 28, 30	0.10	0.10	0.80	113.09	0.0058	176.58
10	7, 11, 13, 15, 28, 30	0.10	0.15	0.75	115.77	0.0047	188.97

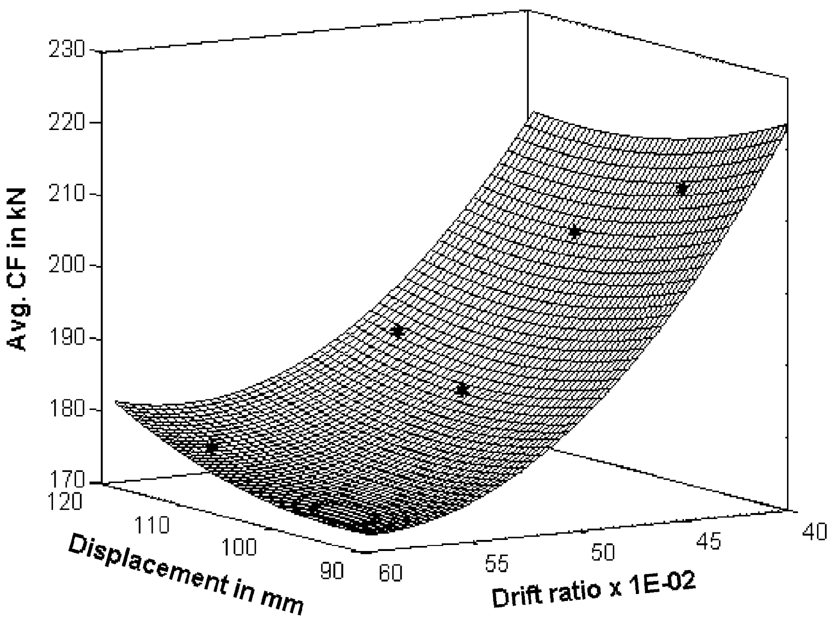

Fig. 10. Pareto optimal curve for optimal positioning of actuators on 10-storey framed structure.

Table 6
Structural parameters for single bay 16-storey framed structure

Floor	Mass (kN)	Stiffness (10^8 N/m)	Damping (kN s/m)
1	6723	2.56	27.0
2–13	5684	2.56	27.0
14–16	5559	1.74	27.0

Table 7
Optimal placement of varied number of actuators using MSGNS algorithm on single bay 16-storey framed structure subjected to El Centro earthquake using four different optimisation criteria

S. no.	Optimisation criterion	Num. of actuators	Optimal locations	Controlled displacement (cm)[a]	Controlled drift ratio[a]
1	Arbitray (top floor)		15, 16	28.44 (28.47)	0.0086 (28.81)
	Displacement		6, 10	26.20 (34.10)	0.0084 (30.46)
	Drift		8, 10	26.85 (32.47)	0.0076 (37.09)
	Abs. accel.	2	4, 14	27.64 (30.48)	0.0077 (36.26)
	Avg. cntl. force		12, 13	28.56 (28.17)	0.0083 (31.29)
2	Arbitray (top floor)		13, 14, 15, 16	26.15 (34.23)	0.0077 (36.26)
	Displacement		5, 8, 10, 16	23.20 (41.65)	0.0074 (38.74)
	Drift		3, 4, 8, 10	25.38 (36.17)	0.0063 (47.85)
	Abs. accel.	4	1, 2, 4, 14	26.29 (33.87)	0.0068 (43.71)
	Avg. cntl. force		12, 14, 15, 16	26.27 (33.92)	0.0079 (34.60)
3	Arbitray (top floor)		11, 12, 13, 14, 15, 16	24.54 (38.27)	0.0071 (41.23)
	Displacement		4, 7, 9, 10, 13, 14	21.02 (47.13)	0.0062 (48.68)
	Drift		1, 3, 4, 7, 8,11	23.99 (39.66)	0.0058 (51.99)
	Abs. accel.	6	1, 2, 4,9, 10, 13	22.75 (42.78)	0.0063 (47.85)
	Avg. cntl. force		1, 12, 13, 14, 15, 16	25.14 (36.77)	0.0071 (41.22)
4	Arbitray (top floor)		9, 10, 11,12, 13,14, 15, 16	23.02 (42.10)	0.0065 (46.19)
	Displacement		4, 7, 8, 10, 11, 12,13, 14	19.53 (50.88)	0.0060 (50.33)
	Drift		1, 3, 4, 7, 8, 10, 11, 14	20.96 (47.28)	0.0054 (55.29)
	Abs. accel.	8	1, 2, 4, 6, 10, 13, 14, 15	21.03 (47.11)	0.0063 (47.84)
	Avg. cntl. force		1, 3, 11, 12, 13, 14, 15, 16	22.84 (42.56)	0.0064 (47.00)
5	Arbitray (top floor)		7,8, 9, 10, 11,12, 13,14, 15, 16	21.20 (46.68)	0.0061 (49.50)
	Displacement		4, 5, 7, 8, 9, 10, 11, 13, 14, 15	18.64 (53.12)	0.0059 (51.16)
	Drift		1, 3, 4, 7, 8, 10, 11, 12, 13, 14	19.08 (52.01)	0.0052 (56.95)
	Abs. accel.	10	1, 2, 4, 6, 9, 10, 11, 12, 13, 14	19.30 (51.45)	0.0059 (51.16)
	Avg. cntl. force		1, 4, 5, 10, 11, 12, 13, 14, 15, 16	21.17 (46.76)	0.0060 (50.33)
6	Arbitray (top floor)		5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16	19.44 (51.11)	0.0059 (51.16)
	Displacement	12	3, 4, 5, 7, 8, 9 10, 11, 12, 13, 14, 15	18.20 (54.23)	0.0054 (55.29)
	Drift		1, 2, 3, 4, 5, 7, 8, 10, 11, 13, 14, 15	18.68 (53.02)	0.0050 (58.60)
	Abs. accel.		1, 2, 4, 6, 9, 10, 11, 12, 13, 14, 15, 16	20.33 (48.87)	0.0058 (51.99)
	Avg. cntl. force		1, 2, 3, 4, 5, 10, 11, 12, 13, 14, 15, 16	20.54 (48.34)	0.0058 (51.99)

[a]The figures given in the parenthesis in columns 5 and 6 of the table indicate the percentage improvement in the displacement and drift ratios, respectively over their uncontrolled parameters.

numbers will coincide with the floor numbers for this problem, as it has only a single bay. A close look at the results furnished in Table 7 clearly indicate that the optimal locations of actuators are distinct for each optimisation criteria and reduction in the controlled parameters is significant when compared to the arbitrary placement of actuators on the top few floors of the structure.

8.7. Influence of detailed FEM models on optimal actuator placement

We have mentioned earlier that it is desirable to use detailed FEM models for solving the optimal placement problem rather than simple shear building models. Here, we demonstrate through numerical examples the significance in using detailed finite element models for optimisation of actuator positioning on tall building frames. While considering detailed FEM models, the framed structures considered as test cases are modelled using a two-node, six degrees-of-freedom planar beam element.

Table 8 shows the optimal positioning of six actuators when shear building model is considered and their corresponding controlled parameters. Each location number given in the table indicates the respective bay in the frame. El-Centro (N–S) earthquake data is considered for this study. The controlled parameters given in Table 8 are computed using detailed finite element analysis considering the position of actuators as dictated by the shear building model. Since, shear building model can only specify the optimal position with respect to the floor, the optimal bay position on the specified floor is arrived by conducting few trials. The optimal positioning of six actuators with detailed finite element models and their respective controlled parameters are also given in Table 8.

As mentioned earlier, the significance of employing detailed finite element models for optimal actuator placement increases with the increase in the number of bays in the building frame. In order to demonstrate this issue, we have considered a five-bay 10-storey framed structure as the second numerical example. Table 9 shows the positioning of six and eight actuators when shear building model of a five-bay 10-storey framed structure given in Fig. 11 and subjected to El-Centre (N–S) earthquake acceleration. The controlled parameters given in the table are computed using detailed finite element analysis considering the position of actuators as dictated by the shear building model. In order to compare the performance of the detailed finite element models, the optimal positions and their corresponding controlled parameters are also given in the table.

A close look at the results presented Tables 8 and 9 clearly indicate that the control performance of the active control system can be significantly improved by considering detailed FEM models together with MSGNS algorithm for optimal placement of specified number of actuators. The variation in the controlled displacement and drift ratio, while using the two different models for three-bay 10-storey structure is found to

Table 8
Optimal actuator locations and their respective peak controlled parameters for the 10-storey three-bay frame model using MSGNS algorithm-shear building model vs detailed finite element model

S. no.	Optimisation criteria	Num. of actuators	Optimal locations	Displacement (cm)	Drift ratio	Absolute acceleration (m/s^2)	Average CF (N) × E05
Shear building model							
1	Controlled displacement	6	12, 15, 18, 24, 27, 30	7.71	0.0049	4.61	1.91
2	Controlled drift ratio	6	6, 9, 12, 15, 21, 30	10.24	0.0041	3.89	1.97
3	Controlled absolute acceleration	6	3, 6, 9, 12, 21, 24	10.75	0.0048	3.43	1.93
4	Avg. CF	6	3, 6, 9, 12, 27, 30	11.14	0.0054	3.72	1.68
Detailed finite element model							
5	Controlled displacement	6	17,20, 23, 26, 29, 30	6.36	0.0046	4.61	1.98
6	Controlled drift ratio	6	8, 11, 15, 18, 24, 27	8.64	0.0035	3.84	2.15
7	Controlled absolute acceleration	6	4, 5, 7, 20, 25, 28	8.65	0.0045	3.03	2.01
8	Avg. CF	6	1, 3, 7, 12, 28, 30	10.68	0.0052	3.40	1.60

Table 9
Optimal actuator locations and their respective peak controlled parameters for the 10-storey five-bay frame model using MSGNS algorithm-shear building model vs detailed finite element model

S. no.	Optimisation criteria	Num. of actuators	Optimal locations	Displacement (cm)	Drift ratio	Absolute acceleration (m/s^2)	Average CF (N) $\times$ E06
Shear building model							
1	Controlled displacement	6	25, 30,35,40,45,50	14.89	0.0076	3.6194	1.4385
2	Controlled drift ratio	6	20, 25, 30, 35, 40, 50	17.24	0.0075	3.6120	1.4878
3	Controlled displacement	8	5, 10, 25, 30, 35, 40, 45, 50	14.86	0.0074	3.588	1.4984
4	Controlled drift ratio	8	15, 20, 25, 30, 35, 40, 45, 50	14.60	0.0067	3.3532	1.6481
Detailed finite element model							
1	Controlled displacement	6	39, 43, 44, 45, 48, 49	10.77	0.0072	3.4035	1.2557
2	Controlled Drift Ratio	6	19, 24, 25, 39, 40, 44	14.64	0.0067	3.5572	1.4755
3	Controlled displacement	8	18, 22, 23, 28, 33, 41, 47, 48	9.62	0.0068	3.4295	1.4778
4	Controlled drift ratio	8	19, 23, 24, 25, 40, 44, 45, 50	11.85	0.0058	3.2724	1.6274

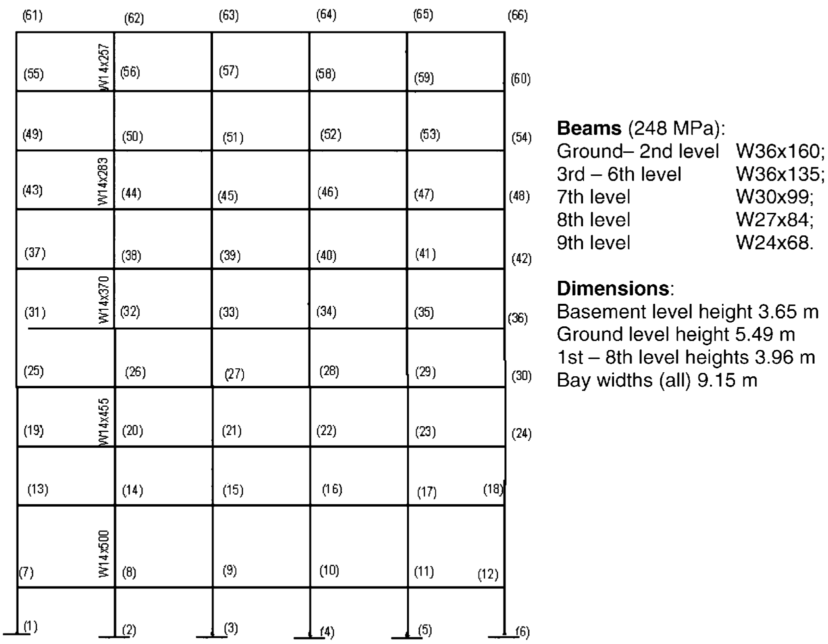

Fig. 11. Ten-storey five-bay framed structure.

be about 20% and 15%, respectively. Similarly, for five-bay 10-storey framed structure, the performance improvement in the peak controlled displacement and drift ratios, with detailed finite element models is about 35% and 18%, respectively. This variation is bound to increase for problems with larger number of bays.

It can be concluded from the results furnished in Tables 8 and 9 that the controlled parameters can be more effectively optimised using detailed finite element models in conjunction with an effective combinatorial optimisation algorithm like the proposed MSGNS algorithm. This optimisation procedure considerably improves the overall performance of the active control system. However, the computational cost is bound to increase with the use of detailed finite element models in the combinatorial optimisation algorithm for function evaluation due to the following:

(i) The search space increases from number of floors to number of floors $\times$ number of bays. This leads to an increase in the number of function evaluations for convergence of the MSGNS algorithm.
(ii) The bandwidth of global stiffness matrix of detailed finite element model and also the total number of degrees of freedom increases substantially when compared to idealised lumped mass model. This result in substantial increase in the cost of each function evaluation.

For the three-bay 10-storey building frame example considered in this paper, the overall computational time required for the MSGNS algorithm with detailed finite element analysis is on an average found to be about 400 times the computational time of the MSGNS algorithm with lumped mass model. The average number of function evaluations required for MSGNS algorithm with detailed finite element models is about 1.8 times higher than the function evaluations required for MSGNS algorithm with shear building models.

In spite of higher computational requirement, use of detailed finite element models in MSGNS algorithm is still recommended in view of the improved control performance as demonstrated in Tables 8 and 9. Further, this exercise is required to be carried out only once during the design stage of a structure before erecting or retrofitting with the active control system. This one time investment in the higher computational cost involved with detailed finite element models during optimisation of actuator placement is negligible, considering the sustained performance gain in the control efficiency of the active control system through out it's life span.

9. Performance evaluation of MSGNS algorithm

Finally, the performance of the proposed MSGNS algorithm is compared with the other popular meta-heuristic algorithms like SA, GA and TS. For this purpose, the same numerical example of 10-storey three-bay framed structure is considered. El-Centro earthquake (with only 10 s of acceleration time history) [31], is considered here primarily to minimise the computer simulation timings during the evaluation of the algorithms.

The population size in GA is taken as 80 with crossover and mutation probabilities as 0.85 and 0.005, respectively. In the present study, tournament selection is employed and the crossover employed is the two-point crossover. Instead of using the conventional mutation operator, we prefer to use the application-specific neighbourhood search algorithms discussed in the paper earlier, as there is ample evidence in the literature that GAs performs much better, if domain knowledge is imparted into the algorithm. Further, to improve the performance, elitism is built into the GA. The solution is assumed to have been converged if there is no improvement in the best-ever solution in the last 30 generations.

The traditional SA algorithm has been employed in the present work for evaluation purpose. The initial temperature has been set as 25 °C. To start with, the temperature decrement operator is taken as 0.98 and later reduced to 0.95 when the current temperature is at 5 °C. The metropolis iterations are set as 50. The solution is assumed to have been converged if the temperature is equal or nearly equal to zero or there is no improvement in the solution in the last 50 evolutions.

The TS algorithm uses the neighbourhood search algorithms discussed in the paper. The tabu tenure is considered as 5. The solution is considered as converged if there is no improvement in the solution in the last 200 iterations. In order to have a fair comparison with MSGNS algorithm, all the performance enhancement features like cache-fetch and *Ricatti-fetch* are built into GA, SA and TS.

Table 10 shows the comparative performances of the four meta-heuristic algorithms considered for evaluation for all the four optimisation criteria. Following are some of the observations made on the results given in Table 10.

(i) The optimal values obtained using the proposed MSGNS algorithm are found to be superior when compared to GA, SA and TS for all the four optimisation criteria considered in this paper.
(ii) The computational performance of the algorithms is measured through the number of function evaluations recorded for each of the algorithm in Table 10. A close look at the results indicates that the proposed MSGNS algorithm is faster when compared to the other three algorithms. While GA and SA take considerable number of function evaluations to converge, TS is relatively faster. However, the optimal results obtained using TS are relatively inferior. It is well known that TS is likely to converge to optimal or near optimal solution rather quickly, when the initial solution chosen (randomly in our case) is close to the optimal solution. In view of this, TS often fails to converge to optimal solution, with the same number of function evaluations, if the same simulation is run repeatedly.
(iii) It can be observed that the difference in the optimal values obtained by each of the algorithm is marginal. However, it is interesting to note that the optimal locations arrived at by these algorithms are distinct. This clearly indicates that several optimal (or near optimal) solutions exist for the optimal actuator placement problem.
(iv) The results presented in Table 10, clearly demonstrates that the proposed MSGNS algorithm built with right balance of diversification and intensification mechanism as discussed earlier, can compute optimal solutions for the optimal placement problem with minimum number of function evaluations.

Table 10
Comparative performance of MSGNS over GA, SA and tabu search

S. no.	Optimisation criteria	Optimisation algorithm	No. of actuators	Optimal locations	Displ. (cm)	Drift ratio	Abs. acln. (m/s^2)	Avg. CF (N) × E05	Function evaluations
1	Controlled displacement	MSGNS	6	17, 20, 23, 26, 29, 30	**6.140**	0.00430	3.321	2.549	518
		SA	6	14, 17, 25, 26, 28, 29	**6.206**	0.00383	3.137	2.549	3825
		GA	6	16, 19, 23, 26, 29, 30	**6.248**	0.00424	3.260	2.513	3840
		Tabu search	6	13, 14, 25, 26, 28, 29	**6.330**	0.00410	3.362	2.556	667
2	Controlled drift ratio	MSGNS	6	11, 15, 18, 20, 24, 27	7.707	**0.00318**	3.809	2.716	460
		SA	6	10, 13, 14, 24, 26, 27	6.924	**0.00318**	3.696	2.824	3490
		GA	6	12, 14, 16, 17, 29, 30	7.170	**0.00320**	3.711	2.696	3680
		Tabu search	6	9, 16, 17, 20, 29, 30	6.820	**0.00350**	3.491	2.660	836
3	Controlled absolute acceleration	MSGNS	6	2, 5, 16, 21, 25, 28	8.179	0.00428	**2.616**	2.455	521
		SA	6	1, 4, 16, 22, 26, 28	8.213	0.00445	**2.631**	2.451	4452
		GA	6	4, 18, 19, 27, 28, 29	7.503	0.00427	**2.629**	2.320	5760
		Tabu search	6	2, 5, 16, 22, 26, 28	8.212	0.00444	**2.632**	2.452	673
4	Controlled avg. CF	MSGNS	6	1, 3, 24, 25, 28, 30	9.076	0.00527	3.405	**2.005**	535
		SA	6	1, 3, 23, 25, 28, 30	8.855	0.00518	3.376	**2.059**	3813
		GA	6	1, 2, 22, 27, 28, 30	9.006	0.00524	3.378	**2.080**	3800
		Tabu search	6	1, 3, 7, 12, 28, 30	10.11	0.00560	3.996	**2.061**	641

10. Conclusions

This paper proposes a MSGNS algorithm for optimal placement of actuators in actively controlled framed structures. The meta-heuristic algorithm employed for the optimal placement problem makes use of the good features of guided search algorithms like SA and TS. The multi-start algorithm overcomes sensitivities of basic SA on cooling schedule. The exploratory search characteristics of the algorithm are also strengthened by embedding a tabu list which records recently visited search patterns and prevents recycling.

Unlike most of the earlier works where simplified structural models have been employed, we propose to employ detailed finite element analysis of framed structures in order to improve the precision in optimal placement of actuators and also to maintain higher degree of accuracy in analysis. A three-bay 10-storey framed structure is considered as a numerical example to demonstrate the effectiveness of the proposed MSGNS algorithm for optimal placement problem. Numerical experiments have also been carried out by varying the number of actuators and also considering four distinct optimisation criteria. Following are some of the conclusions drawn based on the numerical experiments.

(i) The problem of optimal placement is a nonlinear discrete optimisation problem. Hence meta-heuristic techniques are ideal for this class of problems.
(ii) The proposed MSGNS algorithm is found to be effective for solving optimal placement problem.
(iii) Studies indicate that the intensity of earthquake has no bearing on the optimal placement of actuators when peak controlled displacement is considered as the optimisation criteria. However, for other optimisation criteria like peak controlled drift ratio, absolute acceleration and control force, optimal actuator placement is found to be sensitive to earthquake characteristics.
(iv) Numerical studies indicate that it is desirable to employ detailed finite element models to precisely identify the optimal actuator locations. For example, using shear building models, the exact location of the actuator with respect to bay cannot be precisely located and also more than one number of actuator in a particular floor cannot be placed. These limitations add additional constraints to the optimisation problem.
(v) The computational cost of the combinatorial optimisation algorithm with detailed finite element models increases substantially when compared to the convergence of the optimisation algorithm with shear building models. However, this one time investment in the computational cost is fully justified in view of the continuous gain in the control performance of the building frame with active control system through out the life span of the system.
(vi) The results may vary with different design criteria and also with the problem type. The proposed algorithm can however be employed to solve optimal placement problem with any other alternate design criteria.
(vii) Since the optimal actuator location vary with the design criteria and also with the dynamic characteristics of the earthquake, it is desirable to solve the proposed combinatorial problem by simultaneously considering all design criteria and also using ensemble of earthquake acceleration data generated using the site-specific response spectrum. Softwares like SIMQKE [32] can be used for this purpose. One can choose the appropriate solution from the wider choice of non-dominated solutions provided by multi-criteria optimisation technique based on qualitative assessment of the site-specific conditions like importance of the structure, actuator capacity, characteristics of the site-specific earthquake and finally the financial considerations.
(viii) Performance evaluation of the proposed MSGNS algorithm clearly indicate that the proposed meta-heuristic algorithm is superior to other popular meta-heuristic algorithms like GA, SA and TS both in terms of obtaining optimal solution and also in terms of computational performance.

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